Welcome to your Further Mathematics summer task!

The summer task is designed to give you an introduction into the further maths syllabus. It is an opportunity to show off your independent study skills and self-teach yourself the foundations of the complex numbers topic.

The 3 objectives are as follows:

Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).

Add, subtract, multiply and divide complex numbers in the form $x + iy$ with x and y real; understand and use the terms 'real part' and 'imaginary part'.

Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.

You must complete the Complex Numbers Assessment on page 2 and hand it in to your teacher during the first lesson back in September.

Watch the following videos.

http://www.examsolutions.net/tutorials/real-imaginary-numbers/?level=A-Level&board=AQA&module=FP1&topic=1745

http://www.examsolutions.net/tutorials/addition-subtraction-multiplying-complex-numbers-simplifying-powers/?level=A-Level&board=AQA&module=FP1&topic=1745

http://www.examsolutions.net/tutorials/solving-problems-complex-numbers/?level=A-Level&board=AQA&module=FP1&topic=1745

http://www.examsolutions.net/tutorials/square-roots-complex-number/?level=A-Level&board=AQA&module=FP1&topic=1745

Use the pdf attached ('Complex Numbers Textbook extract') which is an extract form a textbook on the topic of complex numbers. There are worked examples you can work through together with additional questions you can use as practice if you wish. All the answers are at the end of the document so you can self-mark any work you do.

Complex Numbers Assessment

Question 1

(a) It is given that $z = x + y\mathrm{i}$, where x and y are real numbers.

(i) Write down, in terms of x and y, an expression for $(z - 2\mathrm{i})^*$. *(1 mark)*

(ii) Solve the equation

$$(z - 2\mathrm{i})^* = 4\mathrm{i}z + 3$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

(b) It is given that $p + q\mathrm{i}$, where p and q are real numbers, is a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$.

Without finding the values of p and q, **state** why $p - q\mathrm{i}$ is **not** a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$. *(1 mark)*

Question 2

2 (a) Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\mathrm{i}$, where p and q are integers. *(3 marks)*

(b) It is given that $z = \mathrm{i}(1 + \mathrm{i})(2 + \mathrm{i})$.

(i) Express z in the form $a + b\mathrm{i}$, where a and b are integers. *(3 marks)*

(ii) Find integers m and n such that $z + mz^* = n\mathrm{i}$. *(3 marks)*

Question 3

3 (a) Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

(i) $x^2 + 9 = 0$; *(1 mark)*

(ii) $(x + 2)^2 + 9 = 0$. *(1 mark)*

(b) (i) Expand $(1 + x)^3$. *(1 mark)*

(ii) Express $(1 + 2\mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(3 marks)*

(iii) Given that $z = 1 + 2\mathrm{i}$, find the value of

$$z^* - z^3$$ *(2 marks)*

Question 4

3 It is given that $z = x + \mathrm{i}y$, where x and y are real.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(z - \mathrm{i})(z^* - \mathrm{i})$$ *(3 marks)*

(b) Given that

$$(z - \mathrm{i})(z^* - \mathrm{i}) = 24 - 8\mathrm{i}$$

find the two possible values of z. *(4 marks)*