

AQA Further Pure 3 Numerical methods

Section 1: Euler's method

Solutions to Exercise

1. (i) $x_0 = 0, y_0 = 1$

$$h = 0.1$$

$$f(x_0, y_0) = x_0 y_0 + y_0^2 = 0 + 1 = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1 \times 1$$

$$= 1.1$$

(ii) $k_1 = hf(x_0, y_0) = 0.1 \times 1 = 0.1$

$$k_2 = hf(x_0 + h, y_0 + h)$$

$$= 0.1f(0.1, 1.1)$$

$$= 0.1(0.1 \times 1.1 + 1.1^2) = 0.132$$

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$$

$$= 1 + \frac{1}{2}(0.1 + 0.132)$$

$$= 1.116$$

2. (i) $x_0 = 1, y_0 = 2$

$$h = 0.1$$

$$f(x_0, y_0) = \frac{x_0^2 - y_0^2}{x_0} = \frac{1 - 4}{1} = -3$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 2 + 0.1 \times -3$$

$$= 1.7$$

(ii) $k_1 = hf(x_0, y_0) = 0.1 \times -3 = -0.3$

$$k_2 = hf(x_0 + h, y_0 + h)$$

$$= 0.1f(1.1, 2.1)$$

$$= 0.1 \left(\frac{1.1^2 - 2.1^2}{1.1} \right) = -0.290909$$

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$$

$$= 2 + \frac{1}{2}(-0.3 - 0.290909)$$

$$= 1.7045$$

AQA FP3 Numerical methods 1 Exercise solutions

3. (i) $x_0 = 0, y_0 = 1$

$$h = 0.1$$

$$f(x_0, y_0) = x_0 y_0 + e^{x_0} = 0 \times 1 + e^0 = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1 \times 1$$

$$= 1.1$$

(ii) $x_1 = 0.1, y_1 = 1.1$

$$y_2 = y_0 + 2hf(x_1, y_1)$$

$$= y_0 + 2h(x_1 y_1 + e^{x_1})$$

$$= 1 + 2 \times 0.1(0.1 \times 1.1 + e^{0.1})$$

$$= 1.2430$$

4. (i) $x_0 = 1, y_0 = 1$

$$h = 0.05$$

$$f(x_0, y_0) = \frac{2x_0 + y_0^2}{x_0^2 + y_0} = \frac{2+1}{1+1} = 1.5$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.05 \times 1.5$$

$$= 1.075$$

(ii) $x_1 = 1.05, y_0 = 1.075$

$$y_2 = y_0 + 2hf(x_1, y_1)$$

$$= y_0 + 2h \left(\frac{2x_1 + y_1^2}{x_1^2 + y_1} \right)$$

$$= 1 + 2 \times 0.05 \left(\frac{2 \times 1.05 + 1.075^2}{1.05^2 + 1.075} \right)$$

$$= 1.0150$$