AQA Further Pure 3 Numerical methods

Section 1: Euler's method

Exercise

1. The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = xy + y^2$

and $y(0) = 1$.

(i) Use the Euler formula $y_{r+1} = y_r + hf(x_r, y_r)$
with $h = 0.1$, to obtain an approximation to $y(0.1)$.

(ii) Use the improved Euler formula $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$
where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + h)$ and $h = 0.1$, to obtain an approximation to $y(0.1)$.

2. The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = \frac{x^2 - y^2}{x}$

and $y(1) = 2$.

(i) Use the Euler formula $y_{r+1} = y_r + hf(x_r, y_r)$
with $h = 0.1$, to obtain an approximation to $y(1.1)$.

(ii) Use the improved Euler formula $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$
where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + h)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$.

3. The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = xy + e^x$

and $y(0) = 1$.

(i) Use the Euler formula $y_{r+1} = y_r + hf(x_r, y_r)$
with $h = 0.1$, to obtain an approximation to $y(0.1)$

(ii) Use the formula $y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$
with your answer to part (i), to obtain an approximation to $y(0.2)$.

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4. The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $\mathrm{f}(x, y) = \dfrac{2x + y^2}{x^2 + y}$

and $y(1) = 1$.

(i) Use the Euler formula $y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$
with $h = 0.05$, to obtain an approximation to $y(1.05)$

(ii) Use the formula $y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$
with your answer to part (i), to obtain an approximation to $y(1.1)$.