
FURTHER MATHS

DRVs and CRVs

Specification content coverage: SA5, SD1, SD6 and SD7

In this test you will be assessed on:

- the expectation of linear functions of discrete random variables
- the probability density function for a continuous random variable
- the cumulative distribution function and its relationship with the probability density function
- the rectangular distribution and its mean and variance

The test comprises two sections. The questions in section A will test you on the basics of the topic. Those in section B require a bit more thinking.

Section A: The basics

- 1 (a) The continuous random variable Y has probability density function $f(y)$ given by

$$f(y) = \begin{cases} \frac{2}{21}(6-y) & 1 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Show that for $1 \leq y \leq 4$ the cumulative distribution function, $F(y)$, is given by

$$F(y) = -\frac{1}{21}(y-1)(y-11).$$

[4 marks]

- 1 (b) Find the exact value of $P(Y \leq 3)$.

[2 marks]

- 2 (a) The continuous random variable X has cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{8}(x+3) & -3 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

What is the name of the distribution given to the continuous random variable X ?

[1 mark]

- 2 (b) Find $P(X > 0)$.

[2 marks]

- 2 (c) Find $E(X)$.

[1 mark]

- 2 (d) Find $\text{Var}(X)$.

[1 mark]

Section B: A bit more thinking

- 3 (a) The discrete random variable X has the probability distribution

x	2	3	4	5
$P(X=x)$	0.3	0.25	0.35	0.1

Find $E\left(\frac{1}{X^2}\right)$.

[2 marks]

- 3 (b) Hence find $E\left(3 - \frac{2}{X^2}\right)$.

[2 marks]

- 3 (c) Given that $\text{Var}(X) = \frac{79}{80}$ find $\text{Var}(3 - 2X)$.

[1 mark]

- 4 (a) The random variable Y has the probability distribution

$$f(y) = \begin{cases} \frac{1}{2^y} & y = 2, 3, 4 \\ ke^{-2y} & y \geq 5 \end{cases}$$

Calculate the exact value of k .

[4 marks]

- 4 (b) Sketch the graph of $f(y)$ for $y \geq 5$.

[2 marks]

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- 5 (a)** The time, t , in minutes, that a customer must wait in a queue is modelled by a continuous random variable T with cumulative distribution function $F(t)$ given by

$$F(t) = \begin{cases} 0 & t < 0 \\ \frac{t^3}{120} & 0 \leq t < 4 \\ \frac{t^2}{225} + \frac{104}{225} & 4 \leq t \leq 11 \\ 1 & t > 11 \end{cases}$$

Find the probability density function for T .

[2 marks]

- 5 (b)** Show that $0 \leq Q_2 < 4$.

[2 marks]

- 5 (c)** Hence find the lower quartile of the distribution.

[2 marks]

- 5 (d)** Find to the nearest second, the time within which 70% of customers have been served.

[4 marks]